2.0 Sine and Cosine Law Intro.Sep2020

Math 20 – 2
Outcome: **Apply the Law of Sines to solve problems.**

SKILL: Cross Multiplying to solve.

$\frac{x}{15} = \frac{10}{40}$ ... $10 \times 15 \div 40$

$x = 3.75$

$\frac{12}{18} = \frac{8}{x}$ ... $18 \times 8 \div 12$

$x = 12$

Pythagoras and SOH CAH TOA can only be used on right triangles.

The Sine Law can be used on any triangle, to find a side or an angle.

The Sine Law in any ΔABC:

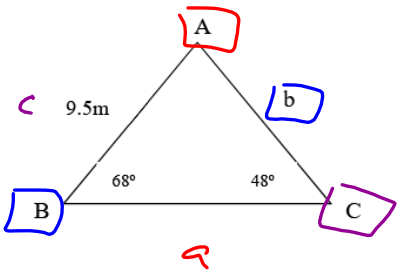
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Steps to solving triangles using the Sine Law:

I. Name all three sides of the triangle by drawing arrows through the angles. The sides are named as lower case letters opposite the angle.
II. Put what you *know* in the formula.
III. Cross multiply. If you're looking for an angle, don't forget to $\sin^{-1}$ to find your answer.

1. Find the missing side or angle indicated.

a) Determine the value of "b", rounded to nearest tenth.



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{a}{\sin A} = \boxed{\frac{b}{\sin 68} = \frac{9.5}{\sin 48}}$

$b = \frac{(0.9272)(9.5)}{0.7431}$

$\frac{b}{0.9272} = \frac{9.5}{0.7431}$

$b = 11.9$

A = 64 a =
B = 68 b = 11.9
C = 48 c = 9.5

$A + 68 + 48 = 180$

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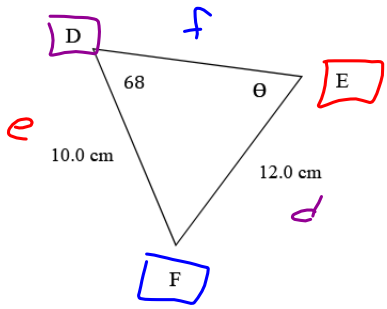
b) Find θ (theta), nearest degree.



$E = 51 \quad e = 10.0$

$D = 68 \quad d = 12.0$

$$\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F}$$

$$\boxed{\frac{12.0}{\sin 68} = \frac{10.0}{\sin E}} = \frac{f}{\sin F}$$

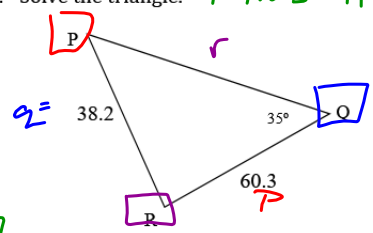
$$\sin E = \frac{(\sin 68)(10.0)}{(12.0)}$$

$$\sin E = 0.7727$$

$$E = \sin^{-1}(0.7727)$$

$$E = 51$$

2. Solve the triangle. FIND ALL ANGLES, SIDES.



I $\boxed{P = 65}$ $\quad p = 60.3$

$Q = 35 \quad q = 38.2$

$\boxed{R = 80}$ II. $\quad \boxed{r = 65.6}$ III.

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{60.3}{\sin P} = \frac{38.2}{\sin 35}$$

$$\sin P = \frac{(60.3)(\sin 35)}{38.2}$$

$$\sin P = 0.9054$$

$$P = \sin^{-1}(0.9054)$$

$$P = 65$$

II.

$R = 180 - 35 - 65$

$R = 80$

III.

$$\frac{r}{\sin 80} = \frac{38.2}{\sin 35} \quad \text{OR} \quad \frac{r}{\sin 80} = \frac{60.3}{\sin 65}$$

$$r = \frac{(\sin 80)(38.2)}{(\sin 35)} = \frac{(0.9848)(38.2)}{(0.5736)}$$

$$r = 65.6$$

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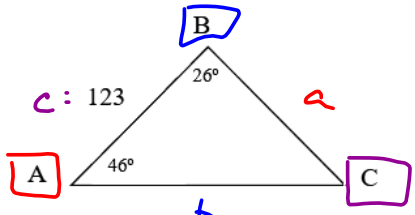
3. Solve the triangle.



$C = 180 - 46 - 26$
$C = 108$

$\frac{a}{\sin 46} = \frac{123}{\sin 108}$ $\frac{a}{0.7193} = \frac{123}{0.9511}$
$a = 93$

$A = 46$ $\boxed{a = 93}$ II.
$B = 26$ $\boxed{b = 56.7}$ II.
$\boxed{C = 108}$ $c = 123$
I.

$\frac{b}{\sin 26} = \frac{123}{\sin 108}$
$b = 56.7$

Outcome: **Apply the Law of Cosines to solve problems.**

What happens if you do NOT have a Right Triangle and do NOT have an angle and side opposite to cross multiply with Law of Sines?

The Cosine Law can be used on any triangle when the Sine Law cannot be used.

- It is used to find an angle in a triangle when you're only given three sides.
- It is used to find the side of a triangle when the known angle is between (subtended by) two known sides.

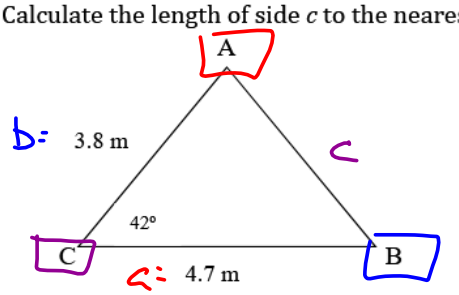
To find a side, use:

$$c^2 = a^2 + b^2 - 2abCosC$$

To find an angle, use:

$$CosC = \frac{a^2 + b^2 - c^2}{2ab}$$

1. Find the indicated side. $c^2 = a^2 + b^2 - 2abCosC$
a) Calculate the length of side c to the nearest tenth of a metre.



$A =$ $a = 4.7$
$B =$ $b = 3.8$
$C = 42$ $\boxed{c = 3.2}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 4.7^2 + 3.8^2 - 2(4.7)(3.8)(\cos 42)$

$\sqrt{c^2} = \sqrt{9.9849}$

$c = 3.2$

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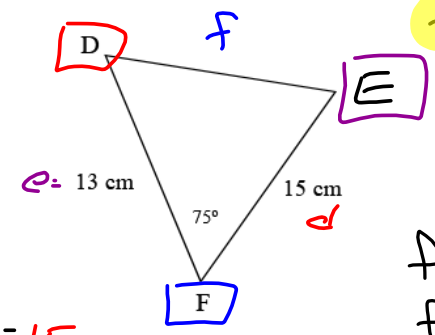
$c^2 = a^2 + b^2 - 2ab\cos C$

b) Calculate the length of side f to the nearest centimeter.

$f^2 = d^2 + e^2 - 2de\cos F$



$f^2 = 15^2 + 13^2 - 2(15)(13)(\cos 75)$

$f^2 = 293.06$

$f = \sqrt{293.06} = 17.119$

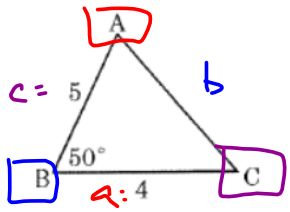
$f = 17$

D = $d = 15$

E = $e = 13$

F = 75 $\boxed{f = 17}$

c) Calculate the length of AC. $\boxed{= b}$



$b^2 = a^2 + c^2 - 2ac\cos B$

$b^2 = 4^2 + 5^2 - 2(4)(5)(\cos 50)$

$b^2 = 15.288$

$b = \sqrt{15.288} = 3.910$

$b = 3.9$

A = $a = 4$

B = 50 $\boxed{b = 3.9}$

C = $c = 5$

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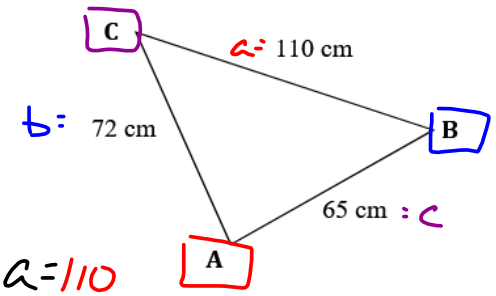
2. Find the indicated angle. $CosC = \dfrac{a^2 + b^2 - c^2}{2ab}$

KNOW ALL SIDES

a) Find the measure of ∠C to the nearest degree.



TOP $= a^2+b^2-c^2$

BOTTOM $= 2ab$

$a^2+b^2-c^2 = 110^2+72^2-65^2$

$2ab = 2(110)(72)$

A:

$\boxed{B = 34}$

C:

$a = 110$

$b = 72$

$c = 65$

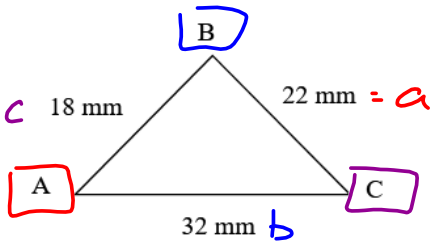
$\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{13059}{15840}$

$\cos C = 0.8244$

$C = \cos^{-1}(0.8244)$

$C = 34.469$

b) Find the measure of the largest angle, rounded to the nearest degree.



$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

A =

$\boxed{B = 106}$

C =

$a = 22$

$b = 32$

$c = 18$

$\cos B = \dfrac{22^2+18^2-32^2}{2(22)(18)} = \dfrac{-216}{792}$

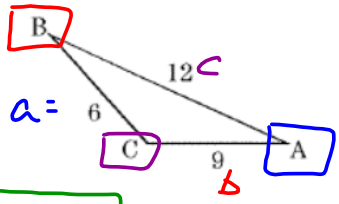
$\cos B = -0.2727$

$B = \cos^{-1}(-0.2727)$

$B = 105.83$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

c) Calculate the measure of ∠A to the nearest degree.



$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos A = \frac{9^2+12^2-6^2}{2(9)(12)} = \frac{189}{216}$$

$$\cos A = 0.875$$

$$A = \cos^{-1}(0.875)$$

$$A = 28.955$$

$\boxed{A = 29 \quad a = 6}$

B = $b = 9$

C = $c = 12$

Law of Sines or Law of Cosines: What do I use?

- If you are able to cross multiply (know or can determine an angle and its opposite side) use the Law of Sines.
- If you have two sides and the included angle, use the Law of Cosines.
- If you have three sides and no angles, use the Law of Cosines.