

**Math 20-2**  
**Radicals with Variables**

**A. Skills:**

Identify square and cube expressions using variables:

$$(5)(5) = 5^2 \text{ so } \sqrt{5^2} = 5$$

$$(x)(x) = x^2 \text{ so } \sqrt{x^2} = x$$

$$(x^2)(x^2) = x^4 \text{ so } \sqrt{x^4} = x^2$$

$$\sqrt[2]{x^2} = (x^2)^{1/2} = x$$

$$\sqrt[3]{\dots} ( )^{1/3}$$

$$(x)(x)(x) = x^3$$

$$\sqrt[3]{x^3} = x$$

**B. Restrictions, domain of square root expressions?**

Examples:

1.  $\sqrt{x-5}$

$x-5$  is positive or zero  
 $x-5$  is zero or more.  
 $x-5=0$   
 $x=5$

$$x \geq 5$$

2.  $\sqrt{2x+1}$

 $2x+1$  is zero or more?

$$\begin{array}{r} 2x+1=0 \\ -1 -1 \\ \hline 2x=0 \\ \hline x=0 \end{array}$$

$$x=-0.5$$

$$x \geq -0.5$$

**C. Express as mixed radicals in simplest form. Restrictions?**

Examples:

1.  $\sqrt{18x^3}$

$$x \geq 0$$

$$\sqrt{9x^2} \sqrt{2x}$$

 $18x^3$  zero or more

$$18x^3=0$$

$$x^3=0$$

"pairs of x"

$$= 3x\sqrt{2x}$$

$$\begin{array}{l} 2. \sqrt{8x^5} \\ \text{zero or more} \\ x \geq 0 \end{array}$$

$$= \sqrt{4x^4} \sqrt{2x}$$

$$= 2x^2 \sqrt{2x}$$

$$\begin{array}{l} 3. 3x\sqrt{49x^7} \\ = 3x \sqrt{49x^6} \sqrt{x} \\ = (3x)(7x^3)\sqrt{x} \\ = 21x^4 \sqrt{x} \end{array}$$

$\sqrt{x^6} = (x^6)^{1/2}$

4.  $\sqrt[3]{24x^7}$

$$= \sqrt[3]{8x^6} \sqrt[3]{3x}$$

$\sqrt[3]{\dots} = ( )^{1/3}$

$$= 2x^2 \sqrt[3]{3x}$$

D. Add and Subtract, Multiply, Divide. Restrictions.

Examples:

$$1. \underline{5\sqrt{x}} + \underline{2\sqrt{x}} = 7\sqrt{x}$$

same size

size doesn't change  
when add

$$2. (2\sqrt{x} + 3)(3\sqrt{x} - 5) = 6\sqrt{x^2} \quad \boxed{-10\sqrt{x} + 9\sqrt{x}} - 15 \\ = 6x - \sqrt{x} - 15$$

$$3. \frac{15\sqrt{6x^3}}{3\sqrt{2x}} = \frac{\cancel{5}\sqrt{6x^3}}{\sqrt{2x}} = \frac{\cancel{5}\sqrt{3x^2}}{(1)}$$

numb: number "3"

radical: radical

" $\sqrt{2x}$ "

$$= 5x\sqrt{3}$$

$$4. \frac{6\sqrt{5}-\sqrt{24x^3}}{2\sqrt{x}} = \frac{\cancel{6}\sqrt{5x} - \cancel{\sqrt{24x^4}}}{2\sqrt{x^2}}$$

numb: numb  
6, -1, 2

5,  $24x^3$ , x

$$= \frac{\cancel{6}\sqrt{5x} - \cancel{\sqrt{4x^4}}\sqrt{6}}{2x}$$

$$= \frac{6\sqrt{5x} - 2x^2\sqrt{6}}{2x}$$

$$= \frac{3\sqrt{5x} - x^2\sqrt{6}}{x}$$

6, -2, 2  
-2