5.11 Z-score problems

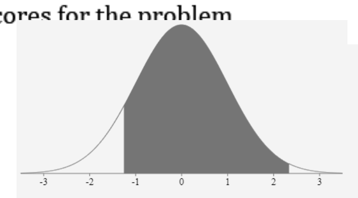
Objective: Problem solve with z-scores.

Skills:

- use a graphing calculator to find the area between two z-scores
  normalcdf(lower-z, upper-z)
- use a graphing calculator to find the z-score knowing the area to the left of a z-score.
  Inverse Normal (area)

Process to find the area (probability)

1. Sketch a normal curve.
2. If not given the z-scores, find the lowest and highest z-scores for the problem.
3. Shade the normal curve between the z-scores.
4. Find the area using normalcdf(__ , __)



−1.25 2.33

Skills – given Z-scores:

- Find the area between z=-1.25 and z=2.33

normalcdf(−1.25, 2.33, 0, 1)

- Find the probability that $z \geq 1.75$ 1.75 lower → 5 upper

normalcdf(1.75, 5)=

5

- Find the probability that $z \leq -1.40$

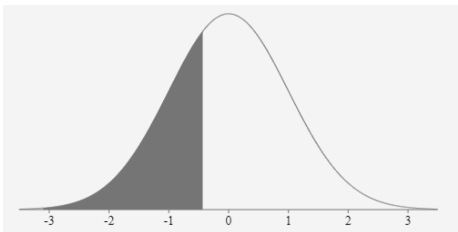
−5 lower ← upper

normalcdf(−5, −1.40)=

−5

Process to find the z-score from area:

1. Sketch a normal curve.
2. Determine the area shaded below the z-score.
3. Find the z-score using invNormal(__).



DISTR

Skills – given area:

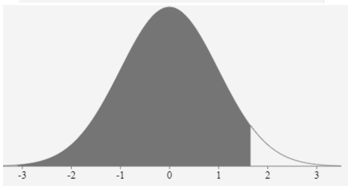
- Find the z-score if the area below z is 0.33

invNorm(0.33) = z = -0.44

- Find the z-score if the area above is 5..0%

5%

LEFT of 5% = 95%



InvNorm(0.95) = z

1.645 = z

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Problem Solving: For each of the following draw and shade a normal curve, find the z-score(s) and/or us z-scores to solve.

1. A study showed that the life of a certain strain of flu is normally distributed with a mean of 12 days and a standard deviation of 3 days. If you caught this type of flu, what is the probability it would last:

   a) 8 days or less?

Find upper and lower z.

Area: normalcdf$(-5, -1.33)$

$z = \frac{x-\mu}{\sigma}$

$z = \frac{8-12}{3} = \frac{-4}{3}$

$z = -1.33$

$-5 \longleftrightarrow -1.33$

   b) 17 days or more?

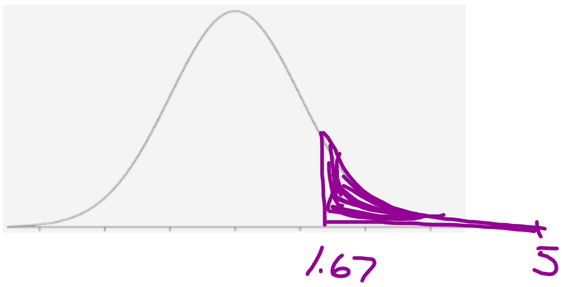
$z = \frac{x-\mu}{\sigma}$

$z = \frac{17-12}{3} = \frac{5}{3} = 1.67$

normalcdf$(1.67, 5)$

$= 0.0475$

OR 4.8%



   c) between 7 and 14 days, inclusive?

$x: 7$

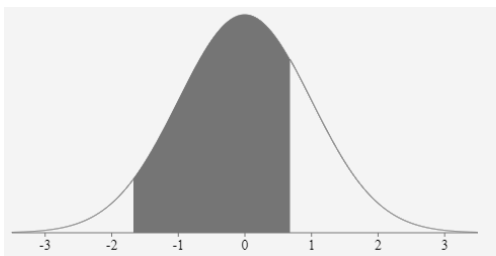
$z = \frac{7-12}{3} = \frac{-5}{3}$

$z = -1.67$

$x = 14$

$z = \frac{14-12}{3} = \frac{2}{3}$

$z = 0.67$



normalcdf$(-1.67, 0.67)$ = 0.7011

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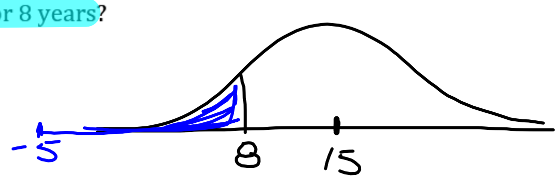
2. After keeping accurate records from the past, a manufacturing company knows their products have an average working period of 15 years, with a standard deviation of 2.4 years. What percentage of the manufacture's products will have to be repaired under warranty if the manufacturer guarantees their product for 8 years?

FIND AREA BELOW "8"



normalcdf (-5, -2.92)

ncd (-5, -2.92)

$$Z = \frac{8-15}{2.4} = \frac{-7}{2.4} = -2.92$$

AREA = 0.0017

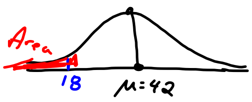
3. A manufacturer of electric shavers has found the average life span of the product is 3.5 years, with a standard deviation of 12 months. An electric shaver that lasts less than 18 months is replaced for free. Assume the distribution of the life span of the shaver is normal.

   a) For every 500 shavers sold, how many can be expected to be replaced for free?

average = 3.5 years

$\mu = 36 + 6$

$\mu = 42$ months



$\mu = 42$
$\sigma = 12$
$x = 18$

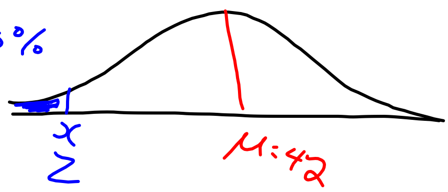
$$z = \frac{18-42}{12} = \frac{-24}{12} = -2$$

normalcdf (-5, -2)
= 0.0227

$$\frac{0.0227}{1} = \frac{x}{500}$$

$x = 11$

   b) A new vice-president of the company does not want to replace more than 1.5% of the shavers. What should the guarantee be on the shavers be?



Z = invNorm (area left)

Z = invNorm (0.015)

Z = -2.17

$$Z = \frac{x-\mu}{\sigma}$$

$$-2.17 = \frac{x-42}{12}$$

$$(-2.17)(12) = x - 42$$

$$26.04 = x - 42$$

$$x = 15.96$$

$x = 16$ months

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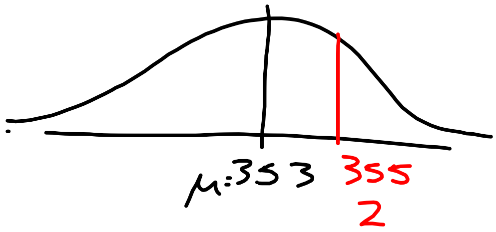
4. The volume of the contents of a soft drink can is normally distributed about a mean of 353 mL, with a standard deviation of 1.5 mL.

   a) Calculate the z-score for a can with a content volume of 355 mL.

$$z = \frac{x - \mu}{\sigma}$$

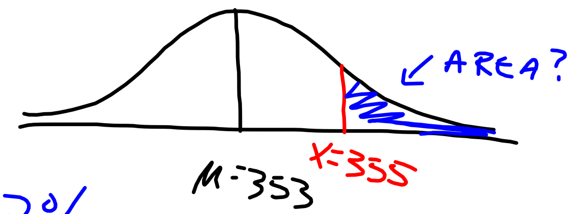
$$z = \frac{355 - 353}{1.5} = \frac{2}{1.5}$$

$$= 1.33$$



   b) What percentage of cans will have content volumes of at least 355 mL?

355 or more...



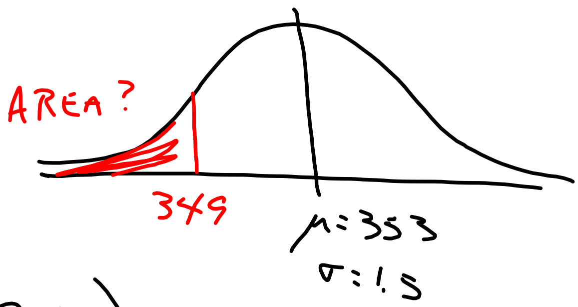
normalcdf
ncd $[1.33, 5]$

$= 0.0918$ or 9.2%

   c) If cans containing 349 mL or less must be rejected, how many cans will be expected to be rejected in a production run of 35 000?

$$z = \frac{349 - 353}{1.5} = \frac{-4}{1.5}$$

$$= -2.67$$



area = normalcdf
ncd $(-5, -2.67)$

AREA = 0.0038

$$\frac{0.0038}{1} = \frac{x}{35000}$$

$x = 133$

5.11 Z-score problems

5. Given $z = \frac{x-\mu}{\sigma}$.

a) The standard deviation for a set of school marks is 8.5.

- What is the average mark for this set of school marks if a mark of 62.0 represents a z-score of -1.40?
- What mark would a student need to be in the top 20% of the school?

$$z = \frac{x-\mu}{\sigma}$$

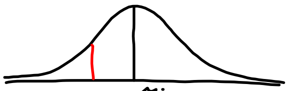
$$-1.40 = \frac{62-\mu}{8.5}$$

$$(-1.40)(8.5) = 62-\mu$$

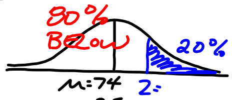
$$-11.9 = 62-\mu$$

$$\mu = 62+11.9$$

$$\mu = 73.9$$



$\mu = 74$



$z = \text{invNorm}(0.80)$

$z = 0.842$

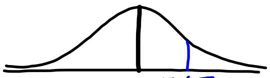
$$z = \frac{x-\mu}{\sigma} \qquad 0.84 = \frac{x-74}{8.5}$$

$$7.14 = x-74$$

$$x = 81$$

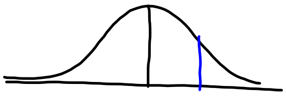
b) A student scored 65 on an exam that had a class average of 56 and a standard deviation of 12.

- Find the z-score for this student.
- Find a new mark for this student if the teacher changes the exam's average to 70 and the standard deviation to 8.0.



$$z = \frac{x-\mu}{\sigma} = \frac{65-56}{12} = \frac{9}{12}$$

$z = 0.75$



$$z = \frac{x-\mu}{\sigma} \qquad 0.75 = \frac{x-70}{8}$$

cross mult

$$6 = x-70$$

add 70

$$\boxed{x = 76}$$