

5.1 Variables

Math 20-2

Radicals with Variables

$$\sqrt[3]{x^6} = (x^6)^{1/3} = x^2$$

$$\sqrt{x^2} = (x^2)^{1/2} = x$$

A. Skills:

Identify square and cube expressions using variables:

$$(5)(5) = 5^2 \text{ so } \sqrt{5^2} = 5$$

$$(x)(x) = x^2 \text{ so } \sqrt{x^2} = x$$

$$(x^2)(x^2) = x^4 \text{ so } \sqrt{x^4} = x^2$$

$$(x^3)(x^3) = x^6 \text{ so } \sqrt{x^6} = x^3$$

$$\sqrt{x^6} = (x^6)^{1/2} = x^3$$

$$(x^2)(x^2)(x^2) = x^6$$

$$(x \cdot x)(x \cdot x)(x \cdot x) = x^6$$

$$\sqrt[3]{x^6} = x^2$$

B. Determine the domain of square root expressions.

- Restrictions: you cannot square root a negative number.
- Find what makes the 'radicand' zero. Then determine the domain for the expression.

Examples:

1. $\sqrt{x-5}$

$$x-5=0$$

$$x=5$$

$$x \geq 5$$

makes zero or positive

2. $\sqrt{2x+1}$

less than -1/2
-2, -5, -10, -100
makes negative

$$2x+1=0$$

$$2x=-1$$

$$x=-1/2$$

more than -1/2
0, 5, 10, 1000
makes positive

domain

$$x \geq -1/2$$

C. Express as mixed radicals in simplest form. Restrictions?

Examples:

1. $\sqrt{18x^3}$

$$= 3x\sqrt{2x}$$

$x \geq 0$

$$\frac{\sqrt{18}}{3\sqrt{2}} \frac{\sqrt{x^3}}{x\sqrt{x}}$$

2. $\sqrt{8x^5}$

$$= 2x^2\sqrt{2x}$$

$x \geq 0$

$$\frac{\sqrt{8}}{2\sqrt{2}} \frac{\sqrt{x^5}}{x^2\sqrt{x}}$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

3. $3x\sqrt{49x^7}$

$x \geq 0$

$$3x\sqrt{49x^6 \cdot x}$$

$$3x(7x^3)\sqrt{x}$$

$$21x^4\sqrt{x}$$

4. $\sqrt[3]{24x^7}$

$$\sqrt[3]{8x^3} \sqrt[3]{x^6 \cdot x}$$

$$= 2\sqrt[3]{3} x^2\sqrt[3]{x}$$

$$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$(x^6)^{1/3} = x^2$$

$$x \in \mathbb{R} = 2x^2\sqrt[3]{3x}$$

$\sqrt[3]{\text{neg}}$ are real numbers...

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D. Add and Subtract, Multiply, Divide. Restrictions.

Examples:

1. $5\sqrt{x} + 2\sqrt{x}$ *same size*

$$= 7\sqrt{x}$$

2. $(2\sqrt{x} + 3)(3\sqrt{x} - 5)$

$$= 6\sqrt{x^2} - 10\sqrt{x} + 9\sqrt{x} - 15$$

$$= 6x - \sqrt{x} - 15$$

$(2\sqrt{x})(3\sqrt{x})$
 $(2\sqrt{x})(-5)$
 $(3)(3\sqrt{x})$
 $(3)(-5)$

3. $\frac{15\sqrt{6x^3}}{3\sqrt{2x}}$

$$= 5\sqrt{3x^2}$$

$$= 5x\sqrt{3}$$

$\frac{15}{3}$ $\frac{\sqrt{6}}{\sqrt{2}}$ $\frac{\sqrt{x^3}}{\sqrt{x}}$

... $\sqrt{x^2}$

4. $\frac{6\sqrt{5} - \sqrt{24x^3}}{2\sqrt{x}}$

6, -1, 2

5, 24x³, x

radical x itself

$$\frac{6\sqrt{5x} - \sqrt{24x^4}}{2\sqrt{x^2}}$$

$$\frac{6\sqrt{5x} - \sqrt{4x^4}\sqrt{6}}{2(x)}$$

$$\frac{6\sqrt{5x} - 2x^2\sqrt{6}}{2x}$$

coefficients
6, 2, 2 ∴ 2

$$\frac{3\sqrt{5x} - x^2\sqrt{6}}{x}$$